Series - Electronic and optical semiconductor devices

Series 13 - Some basic notions on laser diodes II

Temporal behavior and small signal modulation of semiconductor laser diodes

1- In the steady-state, the system of coupled equations must ensure that

$$\begin{cases} \frac{dn}{dt} = 0 \\ \\ \frac{ds}{dt} = 0 \end{cases} \quad \text{i.e.} \quad \begin{aligned} &\frac{J_0}{qd} - \frac{n_0}{\tau} - c'g\, s_0 (n_0 - n_{tr}) = 0 \\ &c g' s_0 (n_0 - n_{tr})\Gamma - \frac{s_0}{\tau_{cav}} = 0 \end{aligned}$$

As a result $c'g s_0 (n_0 - n_{tr}) = \frac{J_0}{qd} - \frac{n_0}{\tau}$ and $\Gamma\left(\frac{J_0}{qd} - \frac{n_0}{\tau}\right) - \frac{s_0}{\tau_{cav}} = 0$

so that $s_0 = \Gamma \tau_{cav}\left(\frac{J_0}{qd} - \frac{n_0}{\tau}\right)$ and $\boxed{n_0 - n_{tr} = \frac{1}{c'g\Gamma\tau_{cav}}}$

$$\boxed{s_0 = \Gamma\tau_{cav}\left(\frac{J_0}{qd} - \frac{n_{tr}}{\tau} - \frac{1}{c'g\Gamma\tau_{cav}\tau}\right)}$$

2- We set $n = n_0 + \delta n$ and $s = s_0 + \delta s$ so that we get:

ⓐ $$\frac{d\delta n}{dt} = \frac{J_0}{qd} + \frac{\delta j}{qd} - \frac{n_0}{\tau} - \frac{\delta n}{\tau} - c'g(s_0 + \delta s)(n_0 + \delta n - n_{tr})$$

i.e. $$\frac{d\delta n}{dt} = \underbrace{\frac{J_0}{qd} - \frac{n_0}{\tau} - c'g s_0 (n_0 - n_{tr})}_{= 0 \text{ (steady-state solution)}} + \frac{\delta j}{qd} - \frac{\delta n}{\tau} - c'g s_0 \delta n - c'g\,\delta s (n_0 - n_{tr}) - \underbrace{c'g\,\delta s\,\delta n}_{\text{2nd order} \hookrightarrow \text{neglected}}$$

so that $$\frac{d\delta n}{dt} = \frac{\delta j}{qd} - \frac{\delta n}{\tau} - c'g\left[s_0 \delta n + \delta s (n_0 - n_{tr})\right]$$

which leads to $$\frac{d\delta n}{dt} = \frac{\delta j}{qd} - \delta n\left[\frac{1}{\tau} + c'g s_0\right] - c'g\,\delta s (n_0 - n_{tr})$$

ⓑ $$\frac{d\delta s}{dt} = c'g(s_0 + \delta s)(n_0 + \delta n - n_{tr})\Gamma - \frac{s_0}{\tau_{cav}} - \frac{\delta s}{\tau_{cav}}$$

i.e. $$\frac{d\delta s}{dt} = \underbrace{c'g s_0 (n_0 - n_{tr})\Gamma - \frac{s_0}{\tau_{cav}}}_{= 0} + \underbrace{\cancel{c'g\,\delta s\,\delta n\,\Gamma}}_{\hookrightarrow \text{neglected}} + c'g\,\delta s (n_0 - n_{tr})\Gamma - \frac{\delta s}{\tau_{cav}} + c'g s_0 \delta n \Gamma$$

so that $$\frac{d\delta s}{dt} = c'g\,\delta s (n_0 - n_{tr})\Gamma - \frac{\delta s}{\tau_{cav}} + c'g s_0 \delta n \Gamma$$

As $n_0 - n_{tr} = \dfrac{1}{c'g\Gamma\tau_{cav}}$ we therefore get:

$$\boxed{\frac{d\delta n}{dt} = \frac{\delta j}{qd} - \delta n\left[\frac{1}{\tau} + c'g s_0\right] - \frac{\delta s}{\Gamma\tau_{cav}}}$$

and $\boxed{\dfrac{d\delta s}{dt} = c'g s_0 \delta n \Gamma}$

3- We make use of the harmonic analysis so that we get:

$$\frac{d^2\delta s}{dt^2} = -\omega^2\delta s = \Gamma c'g s_0 \frac{d\delta n}{dt}$$

i.e. $-\omega^2\delta s = \Gamma c'g s_0 \dfrac{\delta j}{qd} - \Gamma c'g s_0\, \delta n\left[\dfrac{1}{\tau} + c'g s_0\right] - \dfrac{\delta s}{\Gamma\tau_{cav}} \times \Gamma c'g s_0$

so that $-\omega^2\delta s = \Gamma c'g s_0\left[\dfrac{\delta j}{qd} - \left[\dfrac{1}{\tau} + c'g s_0\right]\dfrac{1}{\Gamma c'g s_0} \times i\omega\delta s\right] - \dfrac{\delta s\, c'g s_0}{\tau_{cav}}$

which leads to $\left[-\omega^2 + \dfrac{c'g s_0}{\tau_{cav}} + i\omega\left[\dfrac{1}{\tau} + c'g s_0\right]\right]\dfrac{\delta s}{s_0} = \Gamma c'g\,(\delta j/qd)$

and finally $\boxed{\dfrac{\delta s(\omega)}{s_0} = \dfrac{\Gamma c'g\,(\delta j/qd)}{-\omega^2 + i\omega\left[1/\tau + c'g s_0\right] + \dfrac{c'g s_0}{\tau_{cav}}}}$

$$|\delta s(\omega)| = \Gamma c'g\,(\delta j/qd) \times \frac{s_0}{\left[\left(-\omega^2 + \frac{c'g s_0}{\tau_{cav}}\right)^2 + \omega^2\left[1/\tau + c'g s_0\right]^2\right]^{1/2}}$$

In addition, $|\delta s(0)| = \Gamma c'g\,(\delta j/qd)\, s_0 \times \dfrac{1}{\dfrac{c'g s_0}{\tau_{cav}}} = \Gamma\tau_{cav}\,(\delta j/qd)$

so that $\boxed{|\delta s(\omega)| = \dfrac{|\delta s(0)|}{\dfrac{\tau_{cav}}{c'g s_0}\left[\left(\omega^2 - \dfrac{c'g s_0}{\tau_{cav}}\right)^2 + \omega^2\left(\dfrac{1}{\tau} + c'g s_0\right)^2\right]^{1/2}}}$

4- The frequency response exhibits a maximum for the oscillation relaxation pulsation ω_R defined by $\dfrac{d|\delta s(\omega)|}{d\omega} = 0$

i.e. we are looking for ω_R such that

$$\frac{d}{d\omega}\left[\left(\omega^2-\frac{c'g s_0}{\tau_{cav}}\right)^2+\omega^2\left(\frac{1}{\tau}+c'g s_0\right)^2\right]=0$$

which means that $2\times 2\omega\left(\omega^2-\frac{c'g s_0}{\tau_{cav}}\right)+2\omega\left(\frac{1}{\tau}+c'g s_0\right)^2=0$

The trivial solution $\omega_R=0$ being discarded we get:

$$2\omega^2-2\frac{c'g s_0}{\tau_{cav}}+\left(\frac{1}{\tau}+c'g s_0\right)^2=0$$

i.e. $$\boxed{\omega_R=\left[\frac{c'g s_0}{\tau_{cav}}-\frac{1}{2}\left(\frac{1}{\tau}+c'g s_0\right)^2\right]^{1/2}\approx\left[\frac{c'g s_0}{\tau_{cav}}\right]^{1/2}}$$

and as $\nu_R=\frac{\omega_R}{2\pi}$ $$\boxed{\nu_R\approx\frac{1}{2\pi}\left[\frac{c'g s_0}{\tau_{cav}}\right]^{1/2}}$$

The physical origin of the oscillation relaxation frequency resides in the energy exchange between electron-hole pair population and photon population which are intimately coupled via the stimulated emission process. More precisely, when the density of electron-hole pairs increases, the gain increases as well as the density of photons. The increase of the density of photons leads to a more efficient stimulated emission process and thus to a decrease of the density of electron-hole pairs (which is depleted by stimulated recombinations), hence the oscillation. The relaxation originates from the loss of photons (via parasitic absorption or through the mirrors), which is described by τ_{cav} in the expression of ω_R/ν_R.

5_ The density of photons s_0 is given by:

$$P_s=\begin{pmatrix}\text{photon}\\ \text{energy}\end{pmatrix}\begin{pmatrix}\text{density of}\\ \text{photons}\end{pmatrix}\begin{pmatrix}\text{effective volume}\\ \text{of the mode}\end{pmatrix}\begin{pmatrix}\text{photon escape}\\ \text{rate}\end{pmatrix}$$

$$= h\nu \qquad s_0 \qquad Lw\underbrace{\frac{d}{\Gamma}}_{=d_{mode}} \qquad c'\alpha_{mirror}$$

where $\alpha_{mirror}=\frac{1}{2L}\ln\frac{1}{R_1R_2}$

i.e. $$s_0=\frac{P_s}{h\nu}\times\frac{1}{AL}\times\frac{n_{sc}}{c}\times 2L\times\left(\ln(1/R_2)\right)^{-1}$$

$$S_0 = \frac{10\times 10^{-3}\times 3.3\times 2}{1.4\times 1.6\times 10^{-19}\times 2\times 10^{-13}\times 3\times 10^{8}\ \ln(1/0.32)} \approx 4.3\times 10^{15}\ \text{cm}^{-3}$$

and $$\nu_R = \frac{1}{2\pi}\left[\frac{(3\times 10^{10})^2}{(3.3)^2}\times\left(10+(2\times 200\times 10^{-4})^{-1}\ln(1/0.32)\right)\times 3\times 10^{-16}\times 4.3\times 10^{15}\right]^{1/2} \approx 10.2\ \text{GHz}$$

6_ The main characteristics of $|\delta s(\omega)|$ is a flat response at small frequencies ($|\delta s(\omega)| \approx |\delta s(0)|$) followed by a peak at ω_R (ν_R) and a strong decrease as ω^{-2}. In addition, the expression of ω_R predicts an increase of S_0 proportional to $S_0^{1/2}$. One of the main applications of semiconductor laser diodes is as a light source for optical telecommunication systems. The higher the modulation bandwidth of such lasers, the larger the bit rate. More information can be obtained in K.Y. Lau et al., Appl. Phys. Lett. 43, 1 (1983).

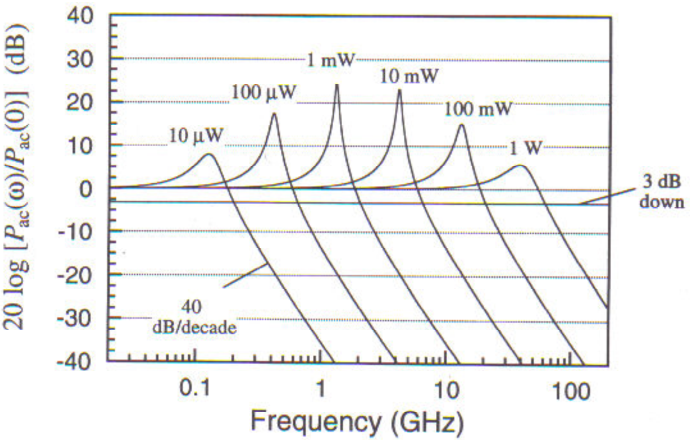

FIGURE 2.12 Frequency response of an idealized diode laser for several different output powers. The active region is characterized by: $h\nu = 1.5$ eV, $a = 5 \times 10^{-16}$ cm^2, $\tau = 3 \times 10^{-9}$ s, $\eta_i = 86.7\%$, and $v_g = 3 \times 10^{10}/4$ cm/s. The laser cavity is characterized by: $\tau_p = 2 \times 10^{-12}$ s (with $\alpha_m = 60$ cm^{-1} and $\alpha_i = 5$ cm^{-1}), $\eta_d = 80\%$, and $V_p = 5$ μm × 0.25 μm × 200 μm. The 20 log [$P_{ac}(\omega)/P_{ac}(0)$] is used because photodetection generates an electrical *current* in direct proportion to the optical *power*. Thus, for a power ratio in the electrical circuit, this current must be squared.